11-07-2018 ①

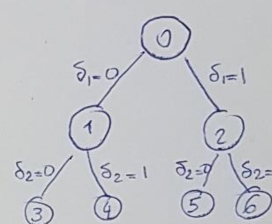
Ex. 2 : solve the following MILP using branch and bound

$$\min_{x_1,\delta_1,\delta_2} \; -\delta_1-\delta_2+x_1$$
$$\delta_1 \leq -0.5\delta_2 + x_1$$
$$\delta_1,\delta_2 \in \{0,1\}$$

$\longrightarrow$ x_1 is not constrained. Needs to be replaced by $x_1^+ - x_1^- = x_1$, $x_1^+, x_1^- \geq 0$

$\downarrow$

$$\min_{x_1^+,x_1^-,\delta_1,\delta_2} \; -\delta_1-\delta_2+x_1^+-x_1^-$$
$$\delta_1+0.5\delta_2-x_1^++x_1^- \leq 0$$
$$\delta_1,\delta_2 \in \{0,1\}$$
$$x_1^+, x_1^- \geq 0$$



$\downarrow$

standard form and relaxation of node 0

$$\min_{x_1^+,x_1^-,\delta_1,\delta_2} \; -\delta_1-\delta_2+x_1^+-x_1^-$$
$$\delta_1+0.5\delta_2-x_1^++x_1^-+s_1=0$$
$$\delta_1+s_2=1$$
$$\delta_2+s_3=1$$
$$x_1^+,x_1^-,\delta_1,\delta_2,s_1,s_2,s_3 \geq 0$$

Phase 1

$$\min \; y_1+y_2+y_3$$
$$\delta_1+0.5\delta_2-x_1^++x_1^-+s_1+y_1=0$$
$$\delta_1+s_2+y_2=1$$
$$\delta_2+s_3+y_3=1$$
$$x_1^+,x_1^-,\delta_1,\delta_2,s_1,s_2,s_3,y_1,y_2,y_3 \geq 0$$

		x_1^+	x_1^-	δ_1	δ_2	s_1	s_2	s_3	y_1	y_2	y_3
	0	0	0	0	0	0	0	0	1	1	1
s_1	0	-1	1	~~-~~1	0.5	1	0	0	1	0	0
s_2	1	0	0	1	0	0	1	0	0	1	0
s_3	1	0	0	0	1	0	0	1	0	0	1

Phase 1 is OK since I have a base which does not include variables y_1, y_2, y_3

Phase 2

		x_1^+	x_1^-	δ_1	δ_2	s_1	s_2	s_3
	0	1	-1	-1	-1	0	0	0
s_1	0	-1	1	1	0.5	1	0	0
s_2	1	0	0	1	0	0	1	0
s_3	1	0	0	0	1	0	0	1

$-(-1)\cdot AUX$

Ratios: 0/1

$\longrightarrow$

ratios

0/0.5

1/1

$\nearrow$

		x_1^+	x_1^-	δ_1	δ_2	s_1	s_2	s_3	
	0	0	0	0	-0.5	1	0	0	$-(-0.5)\cdot AUX$
x_1^-	0	-1	1	1	0.5	1	0	0	$\rightarrow AUX = [0 \mid -2\ 2\ 2\ 1\ 2\ 0$
s_2	1	0	0	1	0	0	1	0	
s_3	1	0	0	0	1	0	0	1	$-AUX$

		x_1^+	x_1^-	δ_1	δ_2	s_1	s_2	s_3	
	0	-1	1	1	0	2	0	0	+AUX
δ_2	0	-2	2	2	1	2	0	0	+2AUX
s_2	1	0	0	1	0	0	1	0	
s_3	1	2	-2	-2	0	-2	0	1	$\Rightarrow$ AUX = [0.5 | 1 -1 -1 0 -1 0 0.5]

ratios 1/2

$\longrightarrow$

		x_1^+	x_1^-	δ_1	δ_2	s_1	s_2	s_3
	0.5	0	0	0	0	1	0	0.5
δ_2	1	0	0	0	1	0	0	1
s_2	1	0	0	1	0	0	1	0
x_1^+	0.5	1	-1	-1	0	-1	0	0.5

2

OK! Phase 2 is over

The optimal solution has cost $J^* = -0.5$ and optimizer
Since the binary variables have values $\delta_1 = 0, \delta_2 = 1$
the obtained solution is the global solution and I do not need to explore the tree any further. Note that the original variable x_1 has value $x_1 = x_1^+ - x_1^- = 0.5$

$$x^* = \begin{bmatrix} x_1^+ \\ x_1^- \\ \delta_1 \\ \delta_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Ex. 3

Solve the problem $J(x_0) = \min_{u_0, u_1, u_2} g_3(x_3) + \sum_{k=0}^{2} g(x_k, u_k)$

Set $J_3(x_3) = g_3(x_3) = \begin{cases} 3 & x_3 = 1 \\ 0 & x_3 = 2 \\ 1 & x_3 = 3 \\ 2 & x_3 = 4 \end{cases} \rightarrow J_2(x_2) = \begin{cases} \min\{\underset{b}{\overset{1\to3}{1+1}}, \underset{c}{\overset{1\to2}{1.5+0}}\} = 1.5 & x_2 = 1 \\ \underset{a}{\overset{2\to3}{1+1}} = 2 & x_2 = 2 \\ \min\{\underset{a}{\overset{3\to2}{0.4+0}}, \underset{b}{\overset{3\to4}{3+2}}\} = 0.4 & x_2 = 3 \\ \min\{\underset{a}{\overset{4\to1}{2+3}}, \underset{c}{\overset{4\to2}{0.5+0}}\} = 0.5 & x_2 = 4 \end{cases}$

$J_1(x_1) = \begin{cases} \min\{\underset{b}{\overset{1\to3}{1+0.4}}, \underset{c}{\overset{1\to2}{1.5+2}}\} = 1.4 & x_1 = 1 \\ \underset{a}{\overset{2\to3}{1+0.4}} = 1.4 & x_1 = 2 \\ \min\{\underset{a}{\overset{3\to2}{0.4+2}}, \underset{b}{\overset{3\to4}{3+0.5}}\} = 2.4 & x_1 = 3 \\ \min\{\underset{a}{\overset{4\to1}{2+1.5}}, \underset{c}{\overset{4\to2}{0.5+2}}\} = 2.5 & x_1 = 4 \end{cases}$

$\mu_2(x_2) = \begin{cases} c & x_2 = 1 \\ a & x_2 = 2 \\ a & x_2 = 3 \\ c & x_2 = 4 \end{cases}$

$\mu_1(x_1) = \begin{cases} b & x_1 = 1 \\ a & x_1 = 2 \\ a & x_1 = 3 \\ c & x_1 = 4 \end{cases}$

$J_0(x_0) = \begin{cases} \min\{\underset{b}{\overset{1\to3}{1+2.4}}, \underset{c}{\overset{1\to2}{1.5+1.4}}\} = 2.9 & x_0 = 1 \\ \underset{a}{\overset{2\to3}{1+2.4}} = 3.4 & x_0 = 2 \\ \min\{\underset{a}{\overset{3\to2}{0.4+1.4}}, \underset{b}{\overset{3\to4}{3+2.5}}\} = 1.8 & x_0 = 3 \\ \min\{\underset{a}{\overset{4\to1}{2+1.4}}, \underset{c}{\overset{4\to2}{0.5+1.4}}\} = 1.9 & x_0 = 4 \end{cases}$

$\mu_0(x_0) = \begin{cases} c & x_0 = 1 \\ a & x_0 = 2 \\ a & x_0 = 3 \\ c & x_0 = 4 \end{cases}$

Compute optimal sequence and cost from $x_0 = 1$

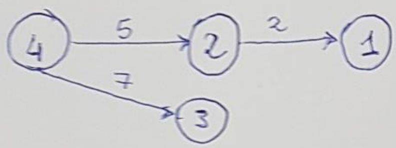
$x_0 = 1 \xrightarrow{\mu_0(1) = c} x_1 = 2 \xrightarrow{\mu_1(2) = a} x_2 = 3 \xrightarrow{\mu_2(3) = a} x_3 = 2$

Ex. 4 Shortest path from vertex 4 to all others

$\bar{v}$	L	$\ell(1)$	$\ell(2)$	$\ell(3)$	$\ell(4)$	$\ell(5)$	$\ell(6)$	
/	$\emptyset$	∞	∞	∞	0	∞	∞	← init
1	$\{4\}$	$10_{(4)}$	$5_{(4)}$	$7_{(4)}$	0	∞	∞	
2	$\{4,2\}$	$7_{(2)}$	5	7	0	∞	∞	
3	$\{4,2,3\}$	7	5	7	0	∞	∞	
4	$\{4,2,3,1\}$	7	5	7	0	∞	∞	

Nodes 5 and 6 cannot be reached from node 4

Graph of shortest paths



pred (4) = ϕ

pred (1) = 2

pred (2) = 4

pred (3) = 4